# Rankine-Hugoniot Spreadsheets

# <u>Users Guide</u>

## <u>Rankine-Hugoniot Equations</u>

Rankine (1870, a & b) and Hugoniot (1884, 1887, 1889) independently solved the equations of conservation of mass, momentum and energy for an ideal gas passing from a supersonic to a subsonic flow, *i. e.* traversing a shock. The resulting equations relate the ratios across the shock of the various physical properties of the flow, *e. g.* the density ratio, $D/D_o$, to the pressure ratio, $P/P_o$, where $D$ and $D_o$, and $P$ and $P_o$, respectively, are the densities and hydrostatic pressures immediately behind a shock and in the ambient gas ahead of the shock. A Galilean transformation provides equations for each of the physical-property ratios in terms of the Mach number of a shock moving into a stationary gas. A more detailed discussion of the Rankine-Hugoniot relationships and their derivations can be found in Ben-Dor *et al* (2001). The equations used in the Dewey McMillin Rankine-Hugoniot spreadsheets were obtained from Gaydon and Hurle (1963), or were derived from those equations.

The set of Rankine-Hugoniot equations relating the physical properties of the gas behind a shock to the shock Mach number are invaluable for those studying shock waves in shock tubes, and blast waves produced by explosions. The validity of the Rankine-Hugoniot equations has been demonstrated beyond question. The results provided by the equations are sufficiently reliable that the preferred method of calibrating transducers, used to measure the physical properties in a shock tube or a blast wave, is to measure the shock's speed, and thus its Mach number, and use that in the equations to determine the required value of the physical property. The derived value can then be related to the output of the transducer.

For strong shocks, additional degrees of freedom of the gas molecules are excited and adjustments must be made to the value of the ratio of specific heats used in the equations. For shocks moving into air, this effect normally is not detectable until the shock speed exceeds about Mach 3, when the error caused by using a constant ratio of specific heats may exceed 1%. Above Mach 5 the error may be as high as 5%. The present spreadsheets assume a constant ratio of specific heats of 1.4. If values are used in the spreadsheets such that the Mach number limits are exceeded, a warning message is displayed.

## <u>Rankine-Hugoniot Spreadsheets</u>

The Rankine-Hugoniot spreadsheets provided by Dewey McMillin & Associates consist of 26 different Excel (©Microsoft Ltd) spreadsheets. Each spreadsheet relates 28 different physical properties behind a shock moving into a stationary ideal gas with a

ratio of specific heats = 1.4 at normal temperature and pressure (NTP), i.e. temperature = 15C = 288.16K, and pressure = 101.325kPa = 14.696psi.

Each row from A5:AG5 to A30:AG30 is a separate spreadsheet that allows the user to input one of the physical properties. The other physical properties corresponding to that value are automatically calculated. For example, consider row A5:AG5, for which the shock Mach number ($M_s$) is the independent variable. If shock Mach number 1.5 is entered in the boxed cell of this row (B5), the physical properties immediately behind the shock are displayed, as shown in figure 1.

A spreadsheet is provided for each of 26 shock properties. The independent variable for each spreadsheet is designated in the first column. Entering a value into the boxed cell of any spreadsheet provides the corresponding values for the other properties. For example, to find the values that correspond to a dynamic pressure of 1.8 atmospheres, enter 1.8 in the boxed cell of row 9 (F9). The other values will be displayed, as shown in figure 2. If a value is entered into a boxed cell such that the resulting shock Mach number exceeds 3.5 or 5.5, an appropriate warning about the increased error will appear in cell B2.

## Physical Properties and Ratios

The following physical properties and ratios are provided by the spreadsheets, listed in the order in which they appear.

$M_s$ - Shock Mach number: the speed of the shock ($V_s$) relative to the speed of sound ($A_o$) in the ambient gas ahead of the shock, $M_s = V_s/A_o$.

$OP/P_o$ - Hydrostatic overpressure ratio ($OP/P_o = P/P_o-1$): the ratio of the hydrostatic overpressure behind the shock (OP) to the pressure in the ambient gas ($P_o$), or hydrostatic overpressure in atmospheres. Overpressure is the pressure in excess of the ambient pressure. Hydrostatic pressure is the pressure measured by a transducer that is side-on to the shock such that the transducer senses no component of the flow produced by the shock. It is the pressure caused by the vibrational motion of the gas molecules, only.

OP(psi) - Hydrostatic overpressure behind the shock in pounds-weight per square inch (psi) for an ambient gas at NTP, $P_o$ = 14.696 psi. To determine the overpressure at any other ambient pressure, multiple the hydrostatic overpressure ratio, $OP/P_o$, from column C by the ambient pressure in psi.

OP(kPa) - Hydrostatic overpressure behind the shock in kiloPascals (kPa) for an ambient gas at NTP, $P_o$ = 101.325kPa. To determine the overpressure at any other ambient pressure, multiple the hydrostatic overpressure ratio, $OP/P_o$, from column C by the ambient pressure in kPa.

$DP/P_o$ - Dynamic pressure ratio: the dynamic pressure behind the shock (DP) relative to the pressure in the ambient gas ($P_o$). Dynamic pressure is a property of the gas

defined as one half the density times the square of the particle speed ($1/2\ D \times U^2$).  N.B.  For a compressible gas, the dynamic pressure is not equal to the stagnation pressure sensed by a pressure transducer face-on to the flow.  See below for Total overpressure.

DP(psi) - Dynamic pressure behind the shock (DP) in pounds-weight per square inch (psi) for a ambient gas at NTP ($P_o = 14.696$psi).  To determine the dynamic pressure at any other ambient pressure, multiply the dynamic pressure ratio from column F by the ambient pressure in psi.  See note above under dynamic pressure ratio.

DP(kPa) - Dynamic pressure behind the shock (DP) in kiloPascals (kPa) for an ambient gas at NTP ($P_o = 101.325$kPa).  To determine the dynamic pressure at any other ambient pressure, multiply the dynamic pressure ratio from column F by the ambient pressure in kPa. See note above under dynamic pressure ratio.

TOP/$P_o$ - Total overpressure (TOP) as a ratio of the pressure in the ambient gas (Po). Total overpressure is the stagnation pressure plus the hydrostatic overpressure.  It is the increase of pressure sensed by a transducer or surface face-on to the flow as a result of the flow being brought to rest adiabatically.  Work is done both to bring the gas to rest and to compress it adiabatically.
N.B.  The Total overpressure is not the same as the reflected overpressure (ROP), for which see below, or the dynamic pressure (DP), see above.
N.B.  It has not been possible to determine an exact analytical expression for the shock Mach number (Ms) in terms of the total overpressure.  An approximate expression has been used, therefore, and for shock Mach numbers from 1.01 to 3.0 the values provided are accurate to within better than 1%.

TOP(psi) - Total overpressure in pounds-weight per square inch (psi) for an ambient gas at NTP, $P_o = 14.696$psi. To determine the total overpressure at any other ambient pressure, multiply the total overpressure ratio from column I by the ambient pressure in psi.  See notes above under total overpressure ratio.

TOP(kPa) - Total overpressure in kiloPascals (kPa) for an ambient gas at NTP, $P_o = 101.345$kPa. To determine the total overpressure at any other ambient pressure, multiply the total overpressure ratio from column I by the ambient pressure in kPa.  See notes above under total overpressure ratio.

ROP/Po - Reflected overpressure (ROP) as a ratio of the ambient pressure ($P_o$).  The reflected overpressure is the increase in pressure on a surface face-on to the shock immediately after the shock is reflected.  This overpressure is maintained until the arrival of the rarefaction wave from the nearest edge of the reflecting surface. The pressure then decays rapidly to the total overpressure (TOP).  The speed of the rarefaction wave is given in columns AB and AC, in ft/s and m/s, respectively. Dividing this speed into the distance from the nearest edge of a structure gives an estimate of the time for which the full reflected pressure will be exerted on the

surface. (The sound speeds behind the reflected shock, in columns AB and AC cannot be used as independent variables in the spreadsheets.)

ROP(psi) - Reflected overpressure (ROP) in pound-weight per square inch (psi) for an ambient gas at NTP, $P_o$ = 14.696psi. To determine the reflected overpressure at any other ambient pressure, multiply the reflected overpressure ratio from column L by the ambient pressure in psi. See notes under reflected overpressure ratio, above.

ROP(kPa) - Reflected overpressure (ROP) in kiloPascals (kPa) for an ambient gas at NTP, $P_o$ = 101.325kPa. To determine the reflected overpressure at any other ambient pressure, multiply the reflected overpressure ratio from column L by the ambient pressure in kPa. See notes under reflected overpressure ratio, above.

D/Do - Density of the gas immediately behind the shock (D) relative to the density of the gas in the ambient atmosphere ($D_o$).

D(lb/ft^3) - Density of the gas immediately behind the shock in pounds per cubic foot for an ambient gas at NTP, $D_o$ = 0.0765lb/ft^3

D(kg/m^3) - Density of the gas immediately behind the shock in kilograms per cubic meter for an ambient gas at NTP, $D_o$ = 1.225kg/m^3.

U/Ao - Speed of the particle flow immediately behind the shock (U) relative to the sound speed in the ambient gas (Ao). N.B. This is not the local Mach number of the flow, for which see below.

U/A - Local Mach number of the flow immediately behind the shock, *i.e.* the particle flow speed (U) relative to be sound speed immediately behind the shock (A). If this value is greater than one, a bow shock will develop around a structure in the flow, and this will modify the total overpressure (TOP), see above. This effect is taken into consideration in calculating the total overpressure in columns I, J and K.

U(ft/s) - Speed of the particle flow immediately behind the shock in feet per second for an ambient gas at NTP, $A_o$ = 1116.878ft/s. To calculate the speed for any other ambient temperature ($T^*$C), multiple the value for U/Ao in column R by the ambient sound speed ($A^*$), where $A^*$ = 1087.42 x sqrt( ($T^*$ + 273.16)/273.16).

U(m/s) - Speed of the particle flow immediately behind the shock in meters per second for an ambient gas at NTP, $A_o$ = 340.4288m/s. To calculate the speed for any other ambient temperature ($T^*$C), multiple the value for U/Ao in column R by the ambient sound speed ($A^*$), where $A^*$ = 331.45 x sqrt( ($T^*$ + 273.16)/273.16).

A/Ao - Speed of sound immediately behind the shock (A) relative to the speed of sound in the ambient gas (Ao).

A(ft/s) - Speed of sound immediately behind the shock (A) in feet per second for an ambient gas at NTP, $A_o = 1116.878$ft/s. To calculate the sound speed for any other ambient temperature ($T^*$C), multiple the value for A/Ao in column V by the ambient sound speed ($A^*$), where $A^* = 1087.42 \times$ sqrt( ($T^* + 273.16$)/273.16).

A(m/s) - Speed of sound immediately behind the shock (A) in meters per second for an ambient gas at NTP, $A_o = 340.4288$m/s. To calculate the sound speed for any other ambient temperature ($T^*$C), multiple the value for A/Ao in column V by the ambient sound speed ($A^*$), where $A^* = 331.45 \times$ sqrt( ($T^* + 273.16$)/273.16).

T/To – Ratio of the absolute temperature of the gas immediately behind the shock (T) to the absolute temperature of the ambient gas ($T_o$).

T(C) - Temperature of the gas immediately behind the shock (T) in Celsius, for an ambient gas at NTP, $T_o = 15$C. To calculate the temperature behind the shock in Celsius for any other ambient temperature ($T^*$) use the temperature ratio (T/To) in column Y in the following formula: $T(C) = (T/To) \times (T^* + 273.16) - 273.16$.

Po/P - Inverse pressure ratio, *viz.* the ratio of the pressure of the ambient gas (Po) to that of the hydrostatic pressure immediately behind the shock (P). Some authors use this ratio to define the shock strength.

## **Manipulation of the spreadsheets**

The spreadsheets can be manipulated easily to allow the user to generate a variety of tables showing all, or a few, of the columns. Most of the cells containing formulae are fixed so that they cannot be accidentally altered. However, the row and column title cells, in column A and rows 3 and 4 are editable, and can be changed by the user. When copying spreadsheets, all the columns to at least column AG must be copied because the cells in columns AD to AG contain intermediate formulae and constants used to calculate the displayed values. Some of the manipulations that can be used are described below.

Selecting and copying a single spreadsheet

If only one the independent variables is of interest, the row for that variable, from column A to at least column AG, can be selected, copied and pasted into a new worksheet. The title rows, 3 and 4, can also be copied and pasted. An example of a spreadsheet created in this way, where dynamic pressure in atmospheres is the variable, is shown as figure2. The pasted spreadsheet in the new worksheet can be replicated as many times as required to produce a table of values.

Creating a limited table of values

The spreadsheets can be used to form tables of values showing a limited number of columns. As an example, a table will be constructed showing the dynamic, total and reflected pressures that correspond to hydrostatic overpressures ranging from 5 to 100 psi.

(a) Select row 7 [OP(psi)], from the A 7 to AG 7, and copy and paste into a new worksheet, in row 5 for example. Also copy the titles in row 3 and 4, from A3 to AC4, and paste into rows 3 and 4 in the new worksheet.

(b) In the new worksheet, copy row 5, from A5 to AG5, and paste into the next 19 rows, i.e. up to row 24.

(c) Enter the value **5** in the boxed cell of row 5, D5. Enter the formula **=D5+5** in cell D6.

(d) Copy D6 and paste into cells D7 to D24. All of the values corresponding to hydrostatic overpressures from 5psi to 100psi should now be displayed. A section of this spreadsheet is shown in figure 3.

(e) Copy columns D, G, J and M, from rows 4 to 24 and paste into a new worksheet using the "Paste Special" and "Values" commands. The resulting table is shown as figure 4, to which column titles have been added, the columns centered and the number of decimal places reduced to two. The table shows the values of the dynamic, total and reflected pressures corresponding to hydrostatic overpressures ranging from 5 to 100psi, at 5psi intervals.

## References

Ben-Dor, G., Igra, O. and Elperin, T., 2001, *Handbook of Shock Waves*, Academic Press, San Diego, USA.

Gaydon, A. G. and Hurle, I. R., 1963, *The Shock Tube in High-Temperature Physics*, Chapman and Hall, London.

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Hugoniot, P. H., 1887, Memoire sur la propagation du mouvement dans les corps et plus spécialement dans les gaz parfaits, 1$^e$ Partie, J. Ecole Polytech. Paris, **57**, 3-97.

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## Figures

|  |  | Hydrostatic Overpressure (OP) | | | Dynamic Pressure (DP) | | |
|---|---|---|---|---|---|---|---|
|  | Ms | Atm | psi | kPa | Atm | psi | kPa |
| Ms | 1.5 | 1.458333 | 21.43167 | 147.7656 | 0.628592 | 9.237787 | 63.69208 |

| Total Overpressure (TOP) | | | Reflected Overpressure (ROP) | | | Density (D) | | |
|---|---|---|---|---|---|---|---|---|
| Atm | psi | kPa | Atm | psi | kPa | D/Do | lb/ft^3 | kg/m^3 |
| 2.146445 | 31.54415 | 217.4885 | 4.425287 | 65.03402 | 448.3922 | 1.862069 | 0.142448 | 2.281034 |

| Flow Speed (U) | | | | Sound Speed (A) | | | Temperature (T) | |
|---|---|---|---|---|---|---|---|---|
| U/Ao | U/A | ft/s | m/s | A/Ao | ft/s | m/s | T/To | T (C) |
| 0.694444 | 0.604387 | 775.6097 | 236.4089 | 1.149007 | 1283.3 | 391.1552 | 1.320216 | 107.2735 |

Fig. 1. Spreadsheet providing the values of various physical properties immediately behind a Mach 1.5 shock in air at NTP

|  |  | Hydrostatic Overpressure (OP) | | | Dynamic Pressure (DP) | | |
|---|---|---|---|---|---|---|---|
|  | Ms | Atm | psi | kPa | Atm | psi | kPa |
| DP/Po | 1.804837 | 2.633675 | 38.70449 | 266.8572 | 1.8 | 26.4528 | 182.385 |

| Total Overpressure (TOP) | | | Reflected Overpressure (ROP) | | | Density (D) | | |
|---|---|---|---|---|---|---|---|---|
| Atm | psi | kPa | Atm | psi | kPa | D/Do | lb/ft^3 | kg/m^3 |
| 4.775054 | 70.17419 | 483.8323 | 9.587351 | 140.8957 | 971.4383 | 2.366911 | 0.181069 | 2.899466 |

| Flow Speed (U) | | | | Sound Speed (A) | | | Temperature (T) | |
|---|---|---|---|---|---|---|---|---|
| U/Ao | U/A | ft/s | m/s | A/Ao | ft/s | m/s | T/To | T (C) |
| 1.042308 | 0.841229 | 1164.131 | 354.8318 | 1.239031 | 1383.846 | 421.802 | 1.535197 | 169.2225 |

Fig. 2. Spreadsheet providing values of various physical properties immediately behind a shock corresponding to a dynamic pressure of 1.8 atmospheres.

| Hydrostatic Overpressure (OP) | | | Dynamic Pressure (DP) | | | Total Overpressure (TOP) | | |
|---|---|---|---|---|---|---|---|---|
| Atm | psi | kPa | Atm | psi | kPa | Atm | psi | kPa |
| 0.340229 | 5 | 34.47367 | 0.039425 | 0.57939 | 3.994742 | 0.38007 | 5.585503 | 38.51055 |
| 0.680457 | 10 | 68.94733 | 0.150714 | 2.214898 | 15.27113 | 0.836061 | 12.28676 | 84.7139 |
| 1.020686 | 15 | 103.421 | 0.324723 | 4.772126 | 32.90253 | 1.364476 | 20.05233 | 138.2555 |
| 1.360915 | 20 | 137.8947 | 0.553794 | 8.138551 | 56.11314 | 1.962669 | 28.84339 | 198.8675 |

Fig. 3. Section of a spreadsheet used to make a table of values related to hydrostatic overpressures at 5psi intervals.

| Hydrostatic Overpressure psi | Dynamic Pressure psi | Total Overpressure psi | Reflected Overpressure psi |
|---|---|---|---|
| 5 | 0.58 | 5.59 | 11.39 |
| 10 | 2.21 | 12.29 | 25.32 |
| 15 | 4.77 | 20.05 | 41.45 |
| 20 | 8.14 | 28.84 | 59.53 |
| 25 | 12.22 | 38.62 | 79.33 |
| 30 | 16.93 | 49.35 | 100.64 |
| 35 | 22.21 | 60.99 | 123.31 |
| 40 | 28.00 | 73.50 | 147.19 |
| 45 | 34.24 | 86.83 | 172.17 |
| 50 | 40.88 | 100.96 | 198.12 |
| 55 | 47.90 | 115.84 | 224.97 |
| 60 | 55.26 | 131.44 | 252.62 |
| 65 | 62.92 | 147.67 | 281.01 |
| 70 | 70.86 | 164.48 | 310.07 |
| 75 | 79.06 | 181.81 | 339.74 |
| 80 | 87.49 | 199.60 | 369.98 |
| 85 | 96.14 | 217.82 | 400.74 |
| 90 | 104.99 | 236.42 | 431.98 |
| 95 | 114.03 | 255.38 | 463.66 |
| 100 | 123.23 | 274.68 | 495.75 |

Fig. 4. Table created from the spreadsheets relating the dynamic, total and reflected pressures to the hydrostatic overpressure from 5 to 100psi at 5psi intervals.

**<u>Request for Acknowledgement</u>**

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